

# Logarithm Function Integration Problem 1

$$\int \frac{\text{Log}[a + b x^m]}{x} dx$$

- *Rubi* uses a simpler rule when a is positive:

$$\text{Int}\left[\frac{\text{Log}[1 + b x^m]}{x}, x\right]$$

$$-\frac{\text{PolyLog}[2, -b x^m]}{m}$$

$$\text{Int}\left[\frac{\text{Log}[5 + b x^m]}{x}, x\right]$$

$$\frac{\text{Log}[5] \text{Log}[x^m]}{m} - \frac{\text{PolyLog}\left[2, -\frac{b x^m}{5}\right]}{m}$$

$$\text{Int}\left[\frac{\text{Log}[a + b x^m]}{x}, x\right]$$

$$\frac{\text{Log}\left[-\frac{b x^m}{a}\right] \text{Log}[a + b x^m]}{m} + \frac{\text{PolyLog}\left[2, 1 + \frac{b x^m}{a}\right]}{m}$$

- *Mathematica* does not use simpler rule when a is positive:

$$\int \frac{\text{Log}[1 + b x^m]}{x} dx$$

$$\frac{\text{Log}[-b x^m] \text{Log}[1 + b x^m]}{m} + \frac{\text{PolyLog}[2, 1 + b x^m]}{m}$$

$$\int \frac{\text{Log}[5 + b x^m]}{x} dx$$

$$\frac{\text{Log}\left[-\frac{b x^m}{5}\right] \text{Log}[5 + b x^m]}{m} + \frac{\text{PolyLog}\left[2, 1 + \frac{b x^m}{5}\right]}{m}$$

$$\int \frac{\text{Log}[a + b x^m]}{x} dx$$

$$\frac{\text{Log}\left[-\frac{b x^m}{a}\right] \text{Log}[a + b x^m]}{m} + \frac{\text{PolyLog}\left[2, 1 + \frac{b x^m}{a}\right]}{m}$$

- *Maple* uses a simpler rule when a is positive:

$$\text{int}(\log(1 + b * x^m) / x, x);$$

$$-\frac{\text{Dilog}[1 + b x^m]}{m}$$

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int (log (5 + b * x^m) / x, x);
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$$\frac{\text{Log}[5] \text{Log}\left[\frac{bx^m}{5}\right]}{m} - \frac{\text{Dilog}\left[1 + \frac{bx^m}{5}\right]}{m}$$

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int (log (a + b * x^m) / x, x);
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$$\frac{\text{Log}\left[-\frac{bx^m}{a}\right] \text{Log}[a + bx^m]}{m} + \frac{\text{Dilog}\left[-\frac{bx^m}{a}\right]}{m}$$

# Logarithm Function Integration Problem 2

$$\int \frac{\text{Log}[a + b x^2]^2}{x^3} dx$$

- The *Rubi* result is relatively simple and free of the imaginary unit:

$$\text{Int}\left[\frac{\text{Log}[a + b x^2]^2}{x^3}, x\right]$$

$$\frac{b \text{Log}\left[-\frac{b x^2}{a}\right] \text{Log}[a + b x^2]}{a} - \frac{(a + b x^2) \text{Log}[a + b x^2]^2}{2 a x^2} + \frac{b \text{PolyLog}\left[2, 1 + \frac{b x^2}{a}\right]}{a}$$

- The *Mathematica* result is complicated and not free of the imaginary unit:

$$\int \frac{\text{Log}[a + b x^2]^2}{x^3} dx$$

$$\begin{aligned} & -\frac{1}{2 a x^2} \left( b x^2 \text{Log}\left[-\frac{i \sqrt{a}}{\sqrt{b}} + x\right]^2 + b x^2 \text{Log}\left[\frac{i \sqrt{a}}{\sqrt{b}} + x\right]^2 + 2 b x^2 \text{Log}\left[-\frac{i \sqrt{a}}{\sqrt{b}} + x\right] \text{Log}\left[\frac{1}{2} - \frac{i \sqrt{b} x}{2 \sqrt{a}}\right] + \right. \\ & 2 b x^2 \text{Log}\left[\frac{i \sqrt{a}}{\sqrt{b}} + x\right] \text{Log}\left[\frac{1}{2} + \frac{i \sqrt{b} x}{2 \sqrt{a}}\right] + 4 b x^2 \text{Log}[x] \text{Log}\left[1 - \frac{i \sqrt{b} x}{\sqrt{a}}\right] + \\ & 4 b x^2 \text{Log}[x] \text{Log}\left[1 + \frac{i \sqrt{b} x}{\sqrt{a}}\right] - 4 b x^2 \text{Log}[x] \text{Log}[a + b x^2] - 2 b x^2 \text{Log}\left[-\frac{i \sqrt{a}}{\sqrt{b}} + x\right] \text{Log}[a + b x^2] - \\ & 2 b x^2 \text{Log}\left[\frac{i \sqrt{a}}{\sqrt{b}} + x\right] \text{Log}[a + b x^2] + a \text{Log}[a + b x^2]^2 + 2 b x^2 \text{Log}[a + b x^2]^2 + 4 b x^2 \text{PolyLog}\left[2, -\frac{i \sqrt{b} x}{\sqrt{a}}\right] + \\ & \left. 4 b x^2 \text{PolyLog}\left[2, \frac{i \sqrt{b} x}{\sqrt{a}}\right] + 2 b x^2 \text{PolyLog}\left[2, \frac{1}{2} - \frac{i \sqrt{b} x}{2 \sqrt{a}}\right] + 2 b x^2 \text{PolyLog}\left[2, \frac{1}{2} + \frac{i \sqrt{b} x}{2 \sqrt{a}}\right] \right) \end{aligned}$$

- *Maple* is unable to integrate the expression:

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int (log (a + b * x^2) ^2 / x^3, x);
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$$\int \frac{\text{Log}[a + b x^2]^2}{x^3} dx$$

# Logarithm Function Integration Problem 3

$$\int \text{Log} \left[ \frac{c x}{a + b x} \right]^3 dx$$

- *Rubi* is able to integrate both special and general cases:

$$\text{Int} \left[ \text{Log} \left[ \frac{x}{a + b x} \right]^3, x \right]$$

$$\frac{(a + b x) \text{Log} \left[ \frac{x}{a + b x} \right]^3}{b} + \frac{3 a \text{Log} \left[ \frac{1}{b + \frac{a}{x}} \right]^2 \text{Log} \left[ -\frac{a}{b x} \right]}{b} - \frac{6 a \text{Log} \left[ \frac{1}{b + \frac{a}{x}} \right] \text{PolyLog} \left[ 2, 1 + \frac{a}{b x} \right]}{b} - \frac{6 a \text{PolyLog} \left[ 3, \frac{b + \frac{a}{x}}{b} \right]}{b}$$

$$\text{Int} \left[ \text{Log} \left[ \frac{c x}{a + b x} \right]^3, x \right]$$

$$\frac{3 a \text{Log} \left[ -\frac{a}{b x} \right] \text{Log} \left[ \frac{c x}{a + b x} \right]^2}{b} + \frac{(a + b x) \text{Log} \left[ \frac{c x}{a + b x} \right]^3}{b} - \frac{6 a \text{Log} \left[ \frac{c x}{a + b x} \right] \text{PolyLog} \left[ 2, 1 + \frac{a}{b x} \right]}{b} - \frac{6 a \text{PolyLog} \left[ 3, 1 + \frac{a}{b x} \right]}{b}$$

- *Mathematica* is only able to integrate the special case:

$$\int \text{Log} \left[ \frac{x}{a + b x} \right]^3 dx$$

$$\frac{3 a \text{Log} \left[ \frac{a}{a + b x} \right] \text{Log} \left[ \frac{x}{a + b x} \right]^2}{b} + x \text{Log} \left[ \frac{x}{a + b x} \right]^3 + \frac{6 a \text{Log} \left[ \frac{x}{a + b x} \right] \text{PolyLog} \left[ 2, \frac{b x}{a + b x} \right]}{b} - \frac{6 a \text{PolyLog} \left[ 3, \frac{b x}{a + b x} \right]}{b}$$

$$\int \text{Log} \left[ \frac{c x}{a + b x} \right]^3 dx$$

$$\int \text{Log} \left[ \frac{c x}{a + b x} \right]^3 dx$$

- *Maple* is unable to integrate either the special or general case:

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int (log (x / (a + b * x)) ^3, x);
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$$\int \text{Log} \left[ \frac{x}{a + b x} \right]^3 dx$$

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int (log (c * x / (a + b * x)) ^3, x);
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$$\int \text{Log} \left[ \frac{c x}{a + b x} \right]^3 dx$$

# Logarithm Function Integration Problem 4

$$\int \text{Log} \left[ \frac{(a + b x)^2}{x^2} \right]^3 dx$$

- *Rubi* is able to integrate the expression:

$$\text{Int} \left[ \text{Log} \left[ \frac{(a + b x)^2}{x^2} \right]^3, x \right]$$

$$\frac{(a + b x) \text{Log} \left[ \left( b + \frac{a}{x} \right)^2 \right]^3}{b} - \frac{6 a \text{Log} \left[ \left( b + \frac{a}{x} \right)^2 \right]^2 \text{Log} \left[ -\frac{a}{b x} \right]}{b} - \frac{24 a \text{Log} \left[ \left( b + \frac{a}{x} \right)^2 \right] \text{PolyLog} \left[ 2, 1 + \frac{a}{b x} \right]}{b} + \frac{48 a \text{PolyLog} \left[ 3, \frac{b + \frac{a}{x}}{b} \right]}{b}$$

- *Mathematica* is unable to integrate the expression:

$$\int \text{Log} \left[ \frac{(a + b x)^2}{x^2} \right]^3 dx$$

$$\int \text{Log} \left[ \frac{(a + b x)^2}{x^2} \right]^3 dx$$

- *Maple* is unable to integrate the expression:

$$\text{int} \left( \ln \left( (a + b x)^2 / x^2 \right)^3, x \right);$$

$$\int \ln \left( \frac{(a + b x)^2}{x^2} \right)^3 dx$$

# Logarithm Function Integration Problem 5

$$\int \frac{1}{x^2 \sqrt{\text{Log}[a x]}} dx$$

- *Rubi* is able to integrate the expression:

$$\text{Int}\left[\frac{1}{x^2 \sqrt{\text{Log}[a x]}}, x\right]$$

$$a \sqrt{\pi} \text{Erf}\left[\sqrt{\text{Log}[a x]}\right]$$

- *Mathematica* is able to integrate the expression:

$$\int \frac{1}{x^2 \sqrt{\text{Log}[a x]}} dx$$

$$a \sqrt{\pi} \text{Erf}\left[\sqrt{\text{Log}[a x]}\right]$$

- *Maple* is unable to integrate the expression:

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int (1 / (x^2 * sqrt (ln (a * x))), x);
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$$\int \frac{1}{x^2 \sqrt{\text{Log}[a x]}} dx$$